



## Course Syllabus

### **Math 2414- Calculus II**

**Catalog Description:** Differentiation and integration of transcendental functions; parametric equations and polar coordinates; techniques of integration; sequences and series; improper integrals.

**Lecture hours = 3, Lab hours = 1**

**Prerequisites:** MATH 2413 – Calculus 1

**Semester Credit Hours:** 4

**Lecture Hours per Week:** 3

**Lab Hours per Week:** 3

**Contact Hours per Semester:** 96

**State Approval Code:** 27.0101.60 19

**Class section meeting time:**

#### **Core Components and Related College Student Learning Outcomes**

This course counts as part of the academic requirements of the Panola College Core Curriculum and an Associate of Arts or Associate of Science degree.  Yes  No: If no, skip to Instructional Goals.

The items below marked with an X reflect the state-mandated outcomes for this course **IF this is a CORE course:**

- Critical Thinking Skills – to include creative thinking, innovation, inquiry and analysis, evaluation and syntheses of information
  - CT1: Generate and communicate ideas by combining, changing, or reapplying existing information
  - CT2: Gather and assess information relevant to a question
  - CT3: Analyze, evaluate, and synthesize information
- Communication Skills – to include effective development, interpretation, and expression of ideas through written, oral, and visual communication
  - CS1: Develop, interpret, and express ideas through written communication
  - CS2: Develop, interpret, and express ideas through oral communication
  - CS3: Develop, interpret, and express ideas through visual communication
- Empirical and Quantitative Skills – to include the manipulation and analysis of numerical data or observable facts resulting in informed conclusions
  - EQS1: Manipulate and analyze numerical data and arrive at an informed conclusion
  - EQS2: Manipulate and analyze observable facts and arrive at an informed conclusion

- Teamwork – to include the ability to consider different points of view and to work effectively with others to support a shared purpose or goal
  - TW1: Integrate different viewpoints as a member of a team
  - TW2: Work with others to support and accomplish a shared goal
- Personal Responsibility – to include the ability to connect choices, actions, and consequences to ethical decision-making
  - PR1: Evaluate choices and actions and relate consequences to decision-making
- Social Responsibility – to include intercultural competence, knowledge of civic responsibility, and the ability to engage effectively in regional, national, and global communities
  - SR1: Demonstrate intercultural competence
  - SR2: Identify civic responsibility
  - SR3: Engage in regional, national, and global communities

**Instructional Goals and Purposes:**

Upon completion of MATH 2414, the student will be able to demonstrate:

1. Competence in finding the antiderivatives of both algebraic and trigonometric functions.
2. Competence in solving applied problems of at least the following types: area, volume, centroids, work, arc length, and liquid pressure.
3. Competence in finding both derivatives and integrals of logarithmic and exponential functions and in applying these concepts to applied problems.
4. Competence in finding both derivatives and integrals of inverse trigonometric functions and solving applied problems using these concepts.
5. Competence in finding both derivatives and integrals of hyperbolic functions and solving applied problems using these concepts.
6. Competence in integrating functions using the standard techniques of integrations
7. Competence in evaluating limits in indeterminate form.
8. Competence in evaluating improper integrals.
9. Competence in using polar coordinates to graph functions and find area of polar curves.
10. Competence in solving problems involving selected topics in solid analytic geometry.
11. Competence in applying the standard tests for convergence/divergence for given infinite series.
12. Competence in integrating and differentiating power series.
13. Competence in determining intervals of convergence for power series.
14. Competence in deriving power series representations of given functions.

## Learning Outcomes:

Upon successful completion of this course, students will:

1. Use the concepts of definite integrals to solve problems involving area, volume, work, and other physical applications.
2. Use substitution, integration by parts, trigonometric substitution, partial fractions, and tables of anti-derivatives to evaluate definite and indefinite integrals.
3. Define an improper integral.
4. Apply the concepts of limits, convergence, and divergence to evaluate some classes of improper integrals.
5. Determine convergence or divergence of sequences and series.
6. Use Taylor and MacLaurin series to represent functions.
7. Use Taylor or MacLaurin series to integrate functions not integrable by conventional methods.
8. Use the concept of polar coordinates to find areas, lengths of curves, and representations of conic sections

## Course Content:

A general description of lecture/discussion topics included in this course are listed in the Learning Objectives / Specific Course Objectives sections of this syllabus.

After studying the material presented in the text(s), lecture, laboratory, computer tutorials, and other resources, the student should be able to complete all behavioral/learning objectives listed below with a minimum competency of 70%.

Upon completion of this section, the student will be able to correctly

1. State the integral definition of the natural logarithmic function.
2. Sketch the graph of the natural logarithmic function  $y = \ln(x)$  and state its domain and range.
3. Differentiate natural log functions using  $(\ln u)' = u^{-1} u'$
4. State and apply the rules (properties) of logarithms.
5. State the relationship between the natural logarithmic and natural exponential function and employ this relationship to convert between the two forms.
6. Sketch the graph of the natural exponential function  $y = e^x$  and state its domain and range.
7. State and apply the laws:  $\ln(e^x) = x$  and  $e^{\ln(x)} = x$ , provided  $x > 0$ .
8. Differentiate the natural exponential composite function  $y = e^u$  where  $u = g(x)$  using the formula  $D_x(e^u) = (e^u)(D_x u)$ .
9. Differentiate natural logarithmic functions of the form  $y = \ln |u|$  where  $u = g(x)$ .
10. Integrate using the formula  $\int du = \ln |u| + C$
11. Integrate using the formula  $\int e^u du = e^u + C$
12. Solve applied problems involving the natural exponential and/or the natural logarithmic functions.

13. Differentiate functions using the technique of logarithmic differentiation.
14. State and apply the definition  $a^u = (e^u)\ln(a)$ .
15. Differentiate using the formula  $D_x (a^u) = a^u [\ln(a)] D_x u$
16. Integrate using the formula  $\int (a^u) du = a^u + C \ln(a)$
17. State and apply the seven (7) properties of exponents.
18. State the definition of the natural number  $e$ .
19. State and apply the definition of inverse function.
20. State and apply the reflective property of inverse functions.
21. State the two conditions that are necessary and sufficient for the inverse of a given function.
22. Find the inverse of a given one-to-one monotonic function.
23. Discuss the continuity and differentiability of an inverse function.
24. State and apply L'Hopital's Rule.
25. State the domain and draw the graph of  $y = \sin^{-1} x$ ,  $y = \cos^{-1} x$  and  $y = \tan^{-1} x$ .
26. State and apply the differentiation formulae for the inverse trigonometric functions.
27. State and apply the integration formulae for the expressions that result in inverse trigonometric functions.
28. State the definition of  $y = \sinh(x)$  and  $y = \cosh(x)$ .
29. Graph  $y = \sinh(x)$  and  $y = \cosh(x)$ .
30. State and apply the differentiation formulae for  $y = \sinh(u)$ ,  $y = \cosh(u)$  and  $y = \tanh(u)$ .
31. State and apply the integration formulae for  $y = \sinh(u)$ ,  $y = \cosh(u)$ , and  $y = \tanh(u)$ .
32. State and apply the formulae to differentiate and integrate the inverse hyperbolic functions.
33. State and apply the formula for arc length:  $s = r\theta$ .
34. State and apply the formula for conversions between degrees and radians.
35. Define the six trigonometric functions.
36. State from memory the selected trigonometric identities given in class.
37. State from memory the sine, cosine, and tangent of the special angles between 0 and 360.
38. Solve trigonometric equations.
39. Sketch from memory the graphs of the six trigonometric functions.
40. State from memory the two special limits (a)  $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$  (b)  $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = 0$
41. Derive and apply the differentiation formulae for the six trigonometric functions.
42. Apply the derivatives of trigonometric functions in extrema and concavity problems.
43. State and apply the basic antiderivative (integration) formulae that follow from the derivatives of the six basic trigonometric functions.
44. Derive and apply the antiderivatives of the six trigonometric functions.
45. Apply the integrals of trigonometric functions to selected applied problems including at least the following:

- a. Area
  - b. volumes of revolution
  - c. work
  - d. center of mass
  - e. average value Integration
46. Perform the following integrations
- a. Integration by parts.
  - b. Integration of powers of sine and cosine.
  - c. Integration of powers of tangent, cotangent, secant, and cosecant.
  - d. Integration via trigonometric substitution.
  - e. Integration of rational algebraic functions by partial fraction expansion when the denominator has only linear factors (both distinct and repeated).
  - f. Integration of rational algebraic function by partial fraction expansion when the denominator has contains irreducible quadratic (and possibly linear) factors.
  - g. Integration of rational functions of sine and cosine.
  - h. Numerical Integrations using the Trapezoid Rule and Simpson's Rule.
  - i. Integrations via the use of a Table of Integrals.
47. State and apply the definition of improper integrals with one or two infinite limits of integration.
48. State and apply the definition of improper integrals with an infinite discontinuity or an interior discontinuity.
49. Apply integration techniques and improper integrals to solve selected applied problems of the types previously detailed.
50. State the definition of an infinite sequence.
51. State the definitions of the limit of a sequence for
- a. a finite limit
  - b. an infinite limit.
52. State the definition of a convergent sequence.
53. State the definition of a divergent sequence.
54. State the following Theorem:
- a.  $\lim r^n = 0$  if  $|r| < 1$
  - b.  $\lim r^n = \infty$  if  $|r| > 1$
55. State the following theorem: If  $\lim f(x) = L$  ( $x \rightarrow \infty$ ) and if  $x$  is defined for every positive integer, then the limit of the sequence  $\{a_n\} = \{f(n)\}$  is also equal to  $L$  ( $x \rightarrow \infty$ ).
56. State the Squeeze Theorem for Infinite Sequences
57. State the definitions of a sequence that is (a) bounded below (b) bounded above (c) bounded.
58. State the definitions of
- a. an upper bound of a sequence  $\{a_n\}$
  - b. a lower bound of a sequence  $\{a_n\}$

- c. a bound of a sequence  $\{a_n\}$ .
59. State the definition of an unbounded sequence.
  60. State the following Theorem: If the sequence  $\{a_n\}$  is convergent, then it is bounded.
  61. State the contrapositive of the above Theorem: Every unbounded sequence is divergent.
  62. State the definitions of sequences that are (a) increasing (b) decreasing (c) monotonic (d) strictly increasing (e) strictly decreasing (f) strictly monotonic
  63. State the following theorem: A bounded monotonic sequence is convergent.
  64. State the Completeness Property.
  65. State the following Theorem: Let  $\{a_n\}$  be a sequence. If  $\lim |a_n| = 0$ , then  $\lim a_n = 0$ .
  66. Write out the first  $n$  terms of a given sequence.
  67. Determine whether a given sequence is convergent or divergent.
  68. Find the limit of a convergent sequence using standard limit techniques.
  69. Find the general term  $a_n$  of a given sequence.
  70. Determine whether a given sequence is bounded or unbounded.
  71. Determine whether a given sequence is increasing, strictly increasing, decreasing, strictly decreasing, or not monotonic.
  72. State the definition of an infinite series.
  73. State the definition of the  $n$ th partial sum of an infinite series.
  74. State the definition of the sequence of partial sums.
  75. State the definitions of (a) a convergent infinite series (b) a divergent infinite series
  76. State the following Theorem: If an infinite series  $\sum a_n$  is convergent then  $\lim a_n = 0$ .
  77. State the contrapositive of the above theorem; i.e., The  $N$ -th Term Divergence Test.
  78. State the Cauchy Criterion for Convergence.
  79. State the definitions of (i) the harmonic series and (ii) the geometric series.
  80. State the conditions for the convergence and divergence of a geometric series.
  81. State and apply the following theorem: If  $\sum a_n$  and  $\sum b_n$  are infinite series such that  $a_i = b_i$  for all  $i > k$ , where  $k$  is a positive integer, then both series converge or both series diverge.
  82. State and apply the following theorem: Let  $c$  be a constant. Suppose that  $\sum a_k$  and  $\sum b_k$  both converge. Then  $\sum (a_k + b_k)$  and  $\sum c a_k$  both converge and (i)  $\sum (a_k + b_k) = \sum a_k + \sum b_k$  and (ii)  $\sum c(a_k) = c[\sum a_k]$
  83. State and apply the following theorem:  
If the series  $\sum a_n$  is convergent and the series  $\sum b_n$  is divergent, then the series  $\sum (a_n + b_n)$  is divergent.
  84. Find the first  $n$  elements of the sequence of partial sums,  $s_n$ , of a given infinite series.
  85. Find a formula for  $s_n$  in terms of  $n$  for a given infinite series.
  86. Determine if a given infinite series is convergent or divergent and, if it is convergent, find its sum.
  87. Write repeating decimals as rational numbers using series techniques.

88. State the following theorem: An infinite series of nonnegative terms is convergent if, and only if, its sequence of partial sums has an upper bound.
89. State and apply the Direct Comparison Test (DCT).
90. State and apply the Limit Comparison Test (LCT).
91. State and apply the MacLaurin-Cauchy Integral Test.
92. State and apply the p-Series Test.
93. State and apply the Ratio Test.
94. State and apply the Root Test.
95. State the following theorem: If  $\sum u_n$  is a given convergent series, of positive terms, the order of the terms can be rearranged, and the resulting series also will be convergent and will have the same sum as the given series.
96. Determine the convergence or divergence of a given series using the above tests.
97. State the definition of absolute convergence.
98. State the following theorem: If  $\sum |a_k|$  converges, then  $\sum a_k$  also converges; that is to say, absolute convergence implies convergence.
99. Identify the converse of the above theorem as being false.
100. State the definition of an alternating series.
101. State the Alternating Series Test (AST).
102. State the definition of a conditionally convergent series.
103. State the following theorem: If  $S = \sum a_k$  is a convergent alternating series with monotone decreasing terms, then for any  $n$   $|S - S_n| < |a_{n+1}|$ .
104. State the following fact: By reordering the terms of a conditionally convergent series, the new series, the new series of rearranged terms can be made to add up to any real number.
105. State the following theorem: Any rearrangement of the terms of an absolutely convergent series converges to the same number.
106. Determine whether a given series is absolutely convergent, conditionally convergent, or divergent.
107. State the definition of a power series in  $x$  and in  $x - x_0$ .
108. State the complete definition of a convergent power series.
109. State the following theorem: (i) If  $\sum (a_k x^k)$  converges at  $x_0$ ,  $x_0 \neq 0$ , then it converges absolutely at all  $x$  such that  $|x| < |x_0|$ . (ii) If  $\sum (a_k x^k)$  diverges at  $x_0$ , then it diverges at all  $x$  such that  $|x| > |x_0|$ .
110. State the definitions of the radius and interval of convergence of a power series.
111. State the following theorem: Consider the power series  $\sum a_k x^k$  and suppose that  $\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n}$  exists and is equal to  $L$ . Then (i) If  $L = k$ , then  $R = 0$ . (ii) If  $L = 0$ , then  $R = \infty$ . (iii) If  $0 < L < 1$ , then  $R = \frac{1}{L}$ .
112. Find the radius of convergence of the interval of convergence of a given power series.
113. State the following theorem: A power series may be differentiated and integrated term-by-term within its radius of convergence.

114. Use a known power series and the above theorem to determine a power series representation for a given series.
115. State the definition of the Taylor Series of a function  $f$  at  $x_0$ .
116. State the definition of the MacLaurin Series of a function  $f$ .
117. State Taylor's Theorem.
118. State Taylor's Formula with remainder.
119. State the definition of an analytic function.
120. State the following theorem: Suppose that the function  $f$  has continuous derivatives of all orders in a neighborhood  $N$  of the number  $x_0$ . Then  $f$  is analytic at  $x_0$  if, and only if,  $\lim_{n \rightarrow \infty} \frac{R_n(x)}{(n+1)!} = 0$  for every  $x$  in  $N$  where  $c_n$  is between  $x_0$  and  $x$ .
121. Find the Taylor (or MacLaurin) Series for a given function. These functions should include such functions as  $e^x$ ,  $\sin(x)$ ,  $\cos(x)$ ,  $\sinh(x)$ ,  $\cosh(x)$ ,  $e^{ax}$ ,  $x^e$ ,  $\sin^2(x)$ ,  $\cos^2(x)$ ,  $\sin^{-1}(x)$ ,  $\cos^{-1}(x)$ , etc.
122. State the Binomial Theorem.
123. Apply the Binomial Theorem to find a MacLaurin Series for a given function.
124. Define the polar coordinate system and locate and identify points in that system.
125. State the relationships between rectangular coordinates of a point and the polar coordinates of a point.
126. Convert the rectangular coordinate representation of a point to the polar coordinate representation and vice versa.
127. Transform rectangular coordinate equations into polar coordinate equations and vice versa.
128. Sketch the graph of a curve expressed as a polar coordinate equation.
129. Recognize and be able to sketch from memory the graphs of special polar coordinate equation forms.
130. Find all the points of intersection of two curves expressed in polar coordinate form by use of both algebraic and graphical methods.
131. Find the length of a curve expressed in polar coordinate form.
132. Find the area of a region bounded by one or more curves expressed in polar coordinate form.
133. Sketch the graph of a curve given the parametric equations which define it.
134. Find a rectangular coordinate equation by eliminating the parameter. Compare the graphs given by the rectangular forms and the parametric form.
135. Find the first, second, and higher ordered derivatives directly from the parametric equations.
136. Find all points of horizontal tangency on the graphs of curves given in parametric form.
137. Find the arc length of curves expressed in parametric form.
138. Find the area of a surface of revolution of a curve defined in parametric form.
139. Evaluate definite integrals of functions defined parametrically. 17. State and apply the tests for symmetry of the graphs of curves defined in polar form.
140. Find the slope of the tangent line to the graph of a curve defined in polar form.
141. Find the area of a surface of revolution of a curve defined in polar form.

**Methods of Instruction/Course Format/Delivery:**

Methods of Instruction/Course Format/Delivery: Methods employed will include Lecture/demonstration, discussion, problem solving, analysis, and reading assignments. Homework will be assigned. Faculty may choose from, but are not limited to, the following methods of instruction:

1. Lecture
2. Discussion
3. Internet
4. Video
5. Television
6. Demonstrations
7. Field trips
8. Collaboration
9. Readings

**Major Assignments/Assessment:**

Faculty may assign both in- and out-of-class activities to evaluate students' knowledge and abilities. Faculty may choose from – but are not limited to -- the following methods attendance, class preparedness and participation. Collaborative learning projects, exams/tests/quizzes, homework, internet, library assignments, readings, research papers, scientific observations, student-teacher conferences, and written assignments.

The Mathematics Department will not accept late work.

**Assessment(s):**

1. Exam per Chapter
2. Comprehensive Final Exam

**Course Grade:**

<b>Assignment Weights</b>	
Class Participation	10%
Homework/Quiz Average	15%
Exams	55%
Comprehensive Final Exam	20%

**Letter Grades for the Course will be assigned as follows:**

- A: 90 < Average < 100
- B: 80 < Average < 90
- C: 70 < Average < 80
- D: 60 < Average < 70

F: 00 < Average < 60

**Texts, Materials, and Supplies:**

- Textbook: Calculus Hybrid , 10<sup>th</sup> Edition Larson
- Webassign Access (Bundled with Textbook)
- Canvas Access
- Scientific Calculator

**Other:**

- For current texts and materials, use the following link to access bookstore listings:  
<http://www.panolacollegestore.com>
- For testing services, use the following link: <http://www.panola.edu/elearning/testing.html>
- If any student in this class has special classroom or testing needs because of a physical learning or emotional condition, please contact the ADA Student Coordinator in Support Services located in the Administration Building or go to <http://www.panola.edu/student-success/disability-support-services/> for more information.
- Withdrawing from a course is the student's responsibility. Students who do not attend class and who do not withdraw will receive the grade earned for the course.
- Student Handbook, *The Pathfinder*: <http://www.panola.edu/student-success/documents/pathfinder.pdf>