Course Syllabus

Math 2320- Differential Equations

Catalog Description: Ordinary differential equations, including linear equations, systems of equations, equations with variable coefficients, existence and uniqueness of solutions, series solutions, singular points, transform methods, and boundary value problems; application of differential equations to real-world problems.

Lecture hours = 3, **Lab hours** = 0

Prerequisites: MATH 2414 – Calculus II

Semester Credit Hours: 3
Lecture Hours per Week: 3
Lab Hours per Week: 0
Contact Hours per Semester: 48
State Approval Code: 27.0101.64 19

Class section meeting time:

Core Components and Related College Student Learning Outcomes
This course counts as part of the academic requirements of the Panola College Core Curriculum and an Associate of Arts or Associate of Science degree. ☐ Yes   ☒No: If no, skip to Instructional Goals.

The items below marked with an X reflect the state-mandated outcomes for this course **IF this is a CORE course**:

☐ Critical Thinking Skills – to include creative thinking, innovation, inquiry and analysis, evaluation and syntheses of information

   ☐ CT1: Generate and communicate ideas by combining, changing, or reapplying existing information

   ☐ CT2: Gather and assess information relevant to a question

   ☐ CT3: Analyze, evaluate, and synthesize information

☐ Communication Skills – to include effective development, interpretation, and expression of ideas through written, oral, and visual communication

   ☐ CS1: Develop, interpret, and express ideas through written communication

   ☐ CS2: Develop, interpret, and express ideas through oral communication

   ☐ CS3: Develop, interpret, and express ideas through visual communication

☐ Empirical and Quantitative Skills – to include the manipulation and analysis of numerical data or observable facts resulting in informed conclusions

   ☐ EQS1: Manipulate and analyze numerical data and arrive at an informed conclusion

☐ EQS2: Manipulate and analyze observable facts and arrive at an informed conclusion

☐ Teamwork – to include the ability to consider different points of view and to work effectively with others to support a shared purpose or goal

    ☐ TW1: Integrate different viewpoints as a member of a team

    ☐ TW2: Work with others to support and accomplish a shared goal

☐ Personal Responsibility – to include the ability to connect choices, actions, and consequences to ethical decision-making

    ☐ PR1: Evaluate choices and actions and relate consequences to decision-making

☐ Social Responsibility – to include intercultural competence, knowledge of civic responsibility, and the ability to engage effectively in regional, national, and global communities

    ☐ SR1: Demonstrate intercultural competence

    ☐ SR2: Identify civic responsibility

    ☐ SR3: Engage in regional, national, and global communities

Instructional Goals and Purposes:

Upon completion of MATH 2320, the student will be able to demonstrate:

1. Competence in classifying differential equations as to ordinary, partial, linear, non-linear, order and degree, and to construct differential equations under given conditions.

2. Competence in solving first order differential equations employing the techniques of variables separable, homogeneous coefficient, or exact equations.

3. Competence in solving applied problems which are linear in form.

4. Competence in solving linear differential equations employing the techniques of integrating factors, substitution, variation of parameters and reduction of order.

5. Competence in finding the Laplace Transform of specified functions and solving linear ordinary differential equations using the Laplace Transform.

Learning Outcomes:

Upon successful completion of this course, students will:

1. Identify homogeneous equations, homogeneous equations with constant coefficients, and exact and linear differential equations.

2. Solve ordinary differential equations and systems of equations using:

    a) Direct integration

    b) Separation of variables

    c) Reduction of order

    d) Methods of undetermined coefficients and variation of parameters

    e) Series solutions

    f) Operator methods for finding particular solutions

    g) Laplace transform methods

3. Determine particular solutions to differential equations with given boundary conditions or initial conditions.

4. Analyze real-world problems in fields such as Biology, Chemistry, Economics, Engineering, and Physics, including problems related to population dynamics, mixtures, growth and decay, heating and cooling, electronic circuits, and Newtonian mechanics.

Course Content:

A general description of lecture/discussion topics included in this course are listed in the Learning Objectives / Specific Course Objectives sections of this syllabus.

After studying the material presented in the text(s), lecture, laboratory, computer tutorials, and other resources, the student should be able to complete all behavioral/learning objectives listed below with a minimum competency of 70%.

Definitions, Families of Curves

1.1 Identify a differential equation (DE).

1.2 Identify the independent variable in a DE. 1.3 Identify the dependent variable in a DE.

2.1 Determine the order of a DE.

2.2 Classify a DE as linear or nonlinear.

2.3 Classify a DE as an ODE or PDE.

3.1 Eliminate the arbitrary constants from a given relation by the generation of an appropriate DE.

4.1 Obtain the DE of a specified family of plane curves and sketch the representative members of the family.

5.1 Optional:

6.1 State the conditions under which a DE has a unique solution.

7.1 Identify and classify, by inspection, an ODE as being separable.

7.2 Obtain the general solution of a separable ODE.

8.1 Identify and classify, by inspection, homogeneous functions.

8.2 Identify the degree of a homogeneous function.

8.3 State the formal definition of homogeneity.

9.1 Identify and classify ODE's with homogeneous coefficients.

9.2 Solve ODE's with homogeneous coefficients by using the appropriate substitution **y = vx** or **x = vy** whichever leads to the more efficient solution.

10.1 State the definition of an exact ODE.

10.2 Test an ODE for exactness.

10.3 Solve an exact ODE.

11.1 State the definition of a first order linear differential equation.

11.2 Write a first order linear ODE in standard form.

11.3 Determine the integrating factor (**IF**) of a first order linear differential equation.

11.4 Solve a first order linear ODE by application of the **IF**.

12.1 Solve for the general solution of a linear ODE.

Elementary Applications Upon completion of this section, the student will be able to correctly formulate the ODE and then solve the resulting equation for *at least* **the following types of elementary applications:**

(1) Growth and decay (i) Population growth (ii) Half-life

(2) Newton's Law of Cooling

(3) An L-R series circuit

(4) A mixture problem

(5) Falling body problems

(6) Frictional forces

(7) Velocity of Escape

(8) Chemical Conversions (Law of Mass Action)

(9) Rates of Dissolving

(10) Logistic Growth and the Price of Commodities

(11) Orthogonal trajectories

Applications of Derivatives Upon completion of this section, the student will be able to correctly

18.1 Identify and state from memory the exact differentials given in the textbook.

18.2 Identify and state from the memory the exact differentials of (a) $d\{\ln(y/x)\}$ (b) $d[\ln(xy)]$ (c) $d[(1/2)\ln(x2 + y2)]$

18.3 Solve ODE's using the above referenced lists of differentials.

18.4 Solve ODE's using an IF of the form $x^k y^n$ to make the given ODE exact.

19.1 Determine the IF (integrating factor) for a first order ODE.

19.2 Apply IF's to convert first order ODE's into exact equations and then solve.

20.1 Solve an ODE of the form **M dx + N dy = 0** by the use of a suitable substitution as suggested by the equation.

21.1 State the general form of Bernoulli's equation.

21.2 State the conditions under which a Bernoulli equation is linear and hence solvable as such.

21.3 Solve Bernoulli equations using the transformation $v = y^{1-n}$ or $z = x^{1-n}$ as appropriate.

22.1 Solve ODE's with coefficients linear in two variable using a transformation of the form **w = ax + by**.

23.1 *Classify and then solve* any of the ODE's in the Miscellaneous Exercise.

Linear Differential Equations Upon completion of this section, the student will be able to correctly 24.1 State the general form of a linear ODE of order n.

24.2 Distinguish between a linear ODE of order n that is homogeneous and one that is non-homogeneous.

24.3 State the Superposition Principle.

25.1 State Theorem 5 of the textbook.

26.1 State the definition of linear independence.

26.2 State the definition of linear dependence.

27.1 State the definition of the Wronskain.

27.2 State the relationship**s** between the vanishing of the Wronskian and dependence and independence.

28.1 State and prove Theorem 7 of the text.

29.1 State the meaning of the phrase "general solution of a non-homogeneous equation." 30.1 State the definition of the differential operator Dk.

31.1 State the fundamental laws of operators with both constant and variable coefficients (primary emphasis being given to constant coefficient operators).

32.1 State and use the exponential shift property of differential operators.

Linear Equations with Constant Coefficients Upon completion of this section, the student will be able to correctly

33.1 Optional.

34.1 Formulate the auxiliary equation of a linear ODE with constant coefficients. 34.2 Use the auxiliary equation to solve a linear ODE with constant coefficients and distinct roots. 35.1 Use the auxiliary equation to solve a linear ODE with constant coefficients and repeated roots or both distinct and repeated roots.

36.1 State Euler's Formulae.

37.1 Use the auxiliary equation to solve a linear ODE with constant coefficients where the auxiliary equation has complex roots and any combination of distinct, repeated, and complex roots.

38.1 State the definitions of $\sinh(x)$ and $\cosh(x)$.

38.2 State the basic properties and identities of the hyperbolic functions.

Non-homogeneous Equations: Undetermined Coefficients Upon completion of this section, the student will be able to correctly

39.1 Construct a linear ODE with real, constant coefficients that is satisfied by a given function.

40.1 Optional.

41.1 Obtain the general solution of an ODE by the

Method of Undetermined Coefficients as outlined in Steps (a) through (d) on page 126 of the text.

41.2. State and apply the statement: The Method of Undetermined Coefficients is applicable only to those ODE's for which $R(x)$ is a solution of a homogeneous linear equation with constant coefficients.

42.1 Obtain a particular solution of a non-homogeneous equation by inspection for the following cases: (a) $R(x) = R_0$, R_0 a constant and $b_n = 0$ (b) $R(x) = R_0$, and D_ky the lowest ordered derivative that actually appears in the ODE. (c) $R(x) = A \sin(Bx)$ and $R(x) = A \cos(Bx)$ (d) $R(x) = e^{ax}$ (e) $R(x)$ is any linear combination of the above functions.

Variation of Parameters Upon completion of this section, the student will be able to correctly

43.1 Optional.

44.1 Solve a non-homogeneous linear equation with constant coefficients by reduction of order.

45.1 Obtain the solution of a non-homogeneous linear equation by **Variation of Parameters.** 46.1 Solve the equation $y'' + y = f(x)$ for unspecified $f(x)$.

46.2 Solve the equation $y'' - y = f(x)$ for unspecified $f(x)$.

46.3 Solve the ODE's in the Miscellaneous Exercise, p. 151.

Inverse Differential Operators Upon completion of this section, the student will be able to correctly

47.1 Use exponential shift to find a particular and general solutions to a particular ODE.

48.1 Optional. 49.1 Evaluate $[1/f(D)]e^{ax}$ and apply to find the particular and general solutions of a given ODE. 49.2 Evaluate $[D^2 + a^2]^{-1} \sin(ax)$ and $[D^2 + a^2]^{-1} \cos(ax)$ and apply to find the particular and general solutions of a given ODE.

The Laplace Transform Upon completion of this section, the student will be able to correctly

60.1 State some elementary transformations including (i) differentiation and (ii) integration. 60.2 State the definition of a linear transformation.

60.3 State the definition of a general linear integral transformation.

60.4 Define the term kernel as applied to transforms.

61.1 State the complete definition of the Laplace transformation.

61.2 Prove that the Laplace transform is a linear transformation.

61.3 State and prove **The First Translation Theorem** (Theorem A of Notes).

61.4 State and prove **The Change of Scale Theorem** (Theorem B of Notes).

62.1 Derive the Laplace transforms of at least the following elementary functions: (a) f(t) = eat (b) f(t) = 1 (c) f(t) = sin (kt) (d) f(t) = cos (kt) (e) f(t) = tn, for n a positive integer. (f) f(t) = sinh (kt) (g) f(t) = cosh (kt)

62.2 State from memory the transform of each of the functions in Objective

62.1 above.

63.1 State the definition of a **sectionally continuous** (*piecewise continuous*) function.

64.1 State the definition of exponential order.

64.2 Prove that a given function is (or is not) of exponential order.

64.3 State and prove Theorem 8 on p. 193 of the text.

64.4 State and prove Theorem 9 on p. 193 of the text.

65.1 State the definition of a function of Class A.

65.2 State the **sufficient conditions** for the Laplace transform of a function to exist.

65.3 State Theorem 10 on p. 195 of the textbook. 65.4 State Theorem 11 on p. 196 of the textbook. Ex.1 State the definition of the **Unit Step Function** (c.f., Section 73 of textbook). Ex.2 State and prove Theorem C at the end of this list of Objectives.

66.1 State and prove Theorem 12 on p. 198 of the textbook.

66.2 State and prove Theorem 13 on p. 198 of the textbook.

66.3 State and prove Theorem 14 on p. 200 of the textbook.

67.1 State and prove Theorem 15 on p. 200 of the textbook.

67.2 State and prove Theorem 16 on p. 200 of the textbook.

67.3 Use the above Theorems to find Laplace transforms.

68.1 State the definition of the Gamma Function.

68.2 State and prove Theorem 17 on p. 202 of the textbook.

68.3 State and prove Theorem 18 on p. 202 of the textbook.

68.4 Evaluate:

68.5 Evaluate: Ex.3 State and prove Theorem D at the end of this list of Objectives. Ex.4 Use Theorem D to find Laplace transforms. Ex.5 State Theorem E at the end of this list of objectives.

69.1 Optional. **Inverse Transforms** Upon completion of this section, the student will be able to correctly

70.1 State the definition of the inverse Laplace transform.

70.2 Write from memory *at least* **the following inverse Laplace transforms: (a) (b) (c) (d) (e) (f)**

70.3 State the conditions under which the inverse Laplace transform exists.

70.4 State Theorem 20 on p. 210 in the textbook.

70.5 State Theorem 21 on p. 211 in the textbook.

70.6 Find the inverse Laplace transform for a F(s).

71.1 Use partial fraction expansions to find inverse Laplace transforms. Ex.6 Use Theorem A and the concept of the inverse Laplace transform to find f(t) given F(s). Ex.7 Use Theorem C and the concept of the inverse Laplace transform to find f(t) given F(s). Ex.8 Use Theorem E (The Convolution Theorem, Section 73 of text) and the concept of the inverse Laplace transform to find f(t) given F(s).

72.1 Use the concept of the inverse Laplace transform to solve initial value problems that are linear ODE's with constant coefficients.

Methods of Instruction/Course Format/Delivery:

Methods of Instruction/Course Format/Delivery: Methods employed will include Lecture/demonstration, discussion, problem solving, analysis, and reading assignments. Homework will be assigned. Faculty may choose from, but are not limited to, the following methods of instruction:

1. Lecture

2. Discussion

3. Internet

4. Video

5. Television

6. Demonstrations

7. Field trips

8. Collaboration

9. Readings

Major Assignments/Assessment:

Faculty may assign both in- and out-of-class activities to evaluate students' knowledge and abilities. Faculty may choose from – but are not limited to -- the following methods attendance, class preparedness and participation. Collaborative learning projects, exams/tests/quizzes, homework, internet, library assignments, readings, research papers, scientific observations, student-teacher conferences, and written assignments.

The Mathematics Department does not accept late work.

Assessment(s):

1. Exam per Chapter

2. Comprehensive Final Exam

Course Grade:

Assignment Weights	
Class Participation	10%
Homework/Quiz Average	15%

Exams   55%
Comprehensive Final Exam   20%

Letter Grades for the Course will be assigned as follows:

A: 90 < Average < 100

B: 80 < Average < 90

C: 70 < Average < 80

D: 60 < Average < 70

F: 00 < Average < 60

Texts, Materials, and Supplies:
- Textbook: Current Textbook
- Online Homework Access
- Canvas Access
- Scientific Calculator

Other:
- For current texts and materials, use the following link to access bookstore listings: http://www.panolacollegestore.com
- For testing services, use the following link: http://www.panola.edu/elearning/testing.html
- If any student in this class has special classroom or testing needs because of a physical learning or emotional condition, please contact the ADA Student Coordinator in Support Services located in the Administration Building or go to http://www.panola.edu/student-success/disability-support-services/ for more information.
- Withdrawing from a course is the student's responsibility. Students who do not attend class and who do not withdraw will receive the grade earned for the course.
- Student Handbook, *The Pathfinder:* http://www.panola.edu/student-success/documents/pathfinder.pdf